

QCD Factorizations in Exclusive $\gamma^*\gamma^* \rightarrow \rho_L^0\rho_L^0$

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Abstract

The exclusive process $e^+e^- \rightarrow e^+e^- \rho_L^0 \rho_L^0$ allows to study various dynamics and factorization properties of perturbative QCD. At moderate energy, we demonstrate how collinear QCD factorization emerges, involving either generalized distribution amplitudes (GDA) or transition distribution amplitudes (TDA). At higher energies, in the Regge limit of QCD, we show that it offers a promising probe of the BFKL resummation effects to be studied at ILC.

1 Introduction: Exclusive processes at high energy QCD

Since a decade, there has been much progress in experimental and theoretical understanding of hard exclusive processes, through the concepts of Generalized Parton Distribution and extensions. Meanwhile, the hard Pomeron [2] concept has been developed and tested for colliders at very large energy. The process

$$\gamma^*\gamma^* \rightarrow \rho_L^0\rho_L^0 \quad (1)$$

with both γ^* hard, in $e^+e^- \rightarrow e^+e^- \rho_L^0\rho_L^0$ with double tagged outgoing leptons, involves several *dynamical regions* (collinear, multiregge) and *factorization* properties of high energy

QCD: it allows a perturbative study of GPD-like objects at moderate s and of the hard Pomeron at asymptotic s . Deeply Virtual Compton Scattering and meson electroproduction on a hadron $\gamma^*h \rightarrow \gamma h, h' h$, as exclusive processes, give access to the full amplitude, which is a convolution, for $-t \ll s$, of a (hard) CF with a (soft) Generalized Parton Distribution [3, 4]. Extensions were made from GPDs. First [3, 5], the crossed process $\gamma^*\gamma \rightarrow hh'$ can be factorized, for $s \ll -t$, as a convolution of a (hard) CF with a (soft) Generalized Distribution Amplitude describing the correlator between two quark fields and a two hadron state. Second [6], starting

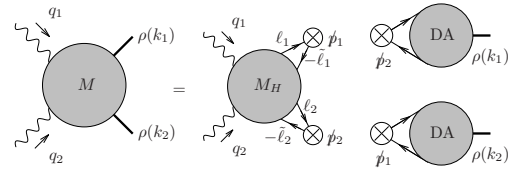


Figure 1: $\gamma^*(Q_1)\gamma^*(Q_2) \rightarrow \rho_L^0(k_1)\rho_L^0(k_2)$ with collinear factorization in $q\bar{q}\rho$ vertices.

from meson electroproduction and performing $t \leftrightarrow u$ crossing, and then allowing the initial and the final hadron to differ, we write the amplitude for the process $\gamma^*h \rightarrow h'' h'$ as a convolution of a (hard) CF with a (soft) Transition Distribution Amplitude describing the $h \rightarrow h'$ transition and with a (soft) Distribution Amplitude (describing $q\bar{q}h''$ vertex). To describe the process (1), we rely

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on collinear factorization at each $q\bar{q}\rho$ vertex only. At high Q_i^2 , outgoing quarks are almost collinear to the ρ mesons, flying in the light cone direction p_i ($i=1$ or 2) and their momenta read $\ell_i \sim z_i k_i$ and $\tilde{\ell}_i \sim \bar{z}_i k_i$. The amplitude M is factorized as a convolution of a hard part M_H with two ρ_L^0 DAs $\phi(z)$ (see Fig.1), defined as matrix elements of non local quarks fields correlator on the light cone (limiting ourselves to longitudinally polarized mesons to avoid potential end-point singularities).

2 Revealing QCD factorization at fixed W^2

2.1 Direct calculation

We compute [7] the amplitude M following the Brodsky, Lepage approach [8], in the forward case for simplicity. It reads^a,

$$M = T^{\mu\nu} \epsilon_\mu(q_1) \epsilon_\nu(q_2), \quad (2)$$

which is expressed as the sum of *two* tensors

$$T^{\mu\nu} = \frac{1}{2} g_T^{\mu\nu} T^{\alpha\beta} g_{T\alpha\beta} + \left(p_1^\mu + \frac{Q_1^2}{s} p_2^\mu \right) \left(p_2^\nu + \frac{Q_2^2}{s} p_1^\nu \right) \frac{4}{s^2} T^{\alpha\beta} p_{2\alpha} p_{1\beta}. \quad (3)$$

In the case of longitudinally polarized photons and at Born order (quark exchange), this results in

$$T^{\alpha\beta} p_{2\alpha} p_{1\beta} = - \frac{s^2 f_\rho^2 C_F e^2 g^2 (Q_u^2 + Q_d^2)}{8 N_c Q_1^2 Q_2^2} \times \int_0^1 dz_1 dz_2 \phi(z_1) \phi(z_2) \left\{ \frac{1}{z_2 \bar{z}_1} + \frac{(1 - \frac{Q_1^2}{s})(1 - \frac{Q_2^2}{s})}{(z_1 + \bar{z}_1 \frac{Q_2^2}{s})(z_2 + \bar{z}_2 \frac{Q_1^2}{s})} + \left(\begin{array}{c} z_1 \leftrightarrow \bar{z}_1 \\ z_2 \leftrightarrow \bar{z}_2 \end{array} \right) \right\}. \quad (4)$$

For transversally polarized photons, one gets

^a $g_T^{\mu\nu} \equiv g^{\mu\nu} - \frac{p_1^\mu p_2^\nu + p_1^\nu p_2^\mu}{p_1 \cdot p_2}$; $s \equiv 2 p_1 \cdot p_2$.

$$T^{\alpha\beta} g_{T\alpha\beta} = - \frac{e^2 (Q_u^2 + Q_d^2) g^2 C_F f_\rho^2}{4 N_c s} \times \int_0^1 dz_1 dz_2 \phi(z_1) \phi(z_2) \left\{ 2 \left(1 - \frac{Q_2^2}{s} \right) \left(1 - \frac{Q_1^2}{s} \right) \times \left[\frac{1}{\left(z_2 + \bar{z}_2 \frac{Q_1^2}{s} \right)^2 \left(z_1 + \bar{z}_1 \frac{Q_2^2}{s} \right)^2} + \left(\begin{array}{c} z_1 \leftrightarrow \bar{z}_1 \\ z_2 \leftrightarrow \bar{z}_2 \end{array} \right) \right] + \left(\frac{1}{\bar{z}_2 z_1} - \frac{1}{\bar{z}_1 z_2} \right) \left[\frac{1}{1 - \frac{Q_2^2}{s}} \left(\frac{1}{\bar{z}_2 + z_2 \frac{Q_1^2}{s}} - \frac{1}{z_2 + \bar{z}_2 \frac{Q_1^2}{s}} \right) - \left(\begin{array}{c} z_1 \leftrightarrow z_2 \\ Q_1 \leftrightarrow Q_2 \end{array} \right) \right] \right\}. \quad (5)$$

The z_i integrations have no end-point singularity ($Q_i^2 \neq 0$ and $\phi(0) = 0$).

2.2 GDA for transverse photon in the limit $\Lambda_{QCD}^2 \ll W^2 \ll \text{Max}(Q_1^2, Q_2^2)$

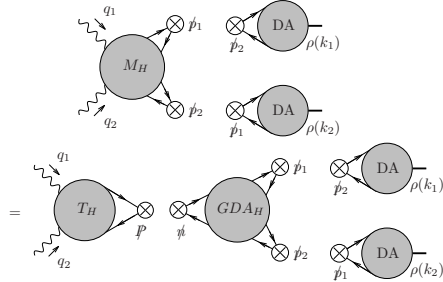


Figure 2: Factorisation of the amplitude in terms of a GDA.

When W^2 is smaller than the highest photon virtuality (for example Q_1^2), (5) simplifies in^b

$$T^{\alpha\beta} g_{T\alpha\beta} \approx \frac{C}{W^2} \int_0^1 dz_1 dz_2 \left(\frac{1}{z_1 + z_1 \frac{Q_2^2}{s}} - \frac{1}{z_1 + \bar{z}_1 \frac{Q_2^2}{s}} \right) \left(\frac{1}{\bar{z}_2 z_1} - \frac{1}{\bar{z}_1 z_2} \right) \phi(z_1) \phi(z_2), \quad (6)$$

showing that the hard amplitude M_H can be factorized as a convolution between a hard

^b We denote $C = \frac{e^2 (Q_u^2 + Q_d^2) g^2 C_F f_\rho^2}{4 N_c}$

coefficient function T_H and a GDA_H , itself *perturbatively* computable (Fig.2), extending the results of [9]. This is proven at Born order. First one computes perturbatively the GDA from its definition as a bilocal correlator: W^2 being hard, the GDA can be factorized as $DA \otimes GDA_H \otimes DA$ (Fig.3). A QCD Wilson line (last term in Fig.3) has to be included to fulfil gauge invariance. It vanishes in forward kinematics. Second, one computes the Born order hard part (Fig.4). These two results combine according to Eq.(6).

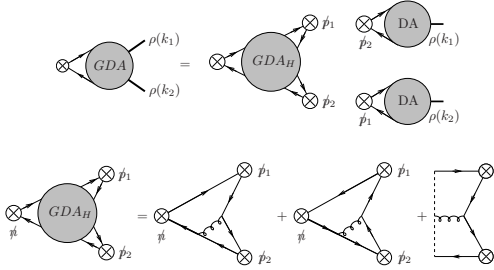


Figure 3: Perturbative GDA factorization.

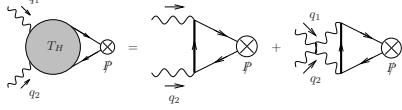


Figure 4: Hard part T_H at lowest order.

2.3 TDA for longitudinal photon in the limit $Q_1^2 \gg Q_2^2$ (or $Q_1^2 \ll Q_2^2$)

The amplitude (5) simplifies in this limit as

$$T^{\alpha\beta} p_{2\alpha} p_{1\beta} = -i \frac{C}{2} \int_{-1}^1 dx \int_0^1 dz_1 \left[\frac{1}{\bar{z}_1(x-\xi)} + \frac{1}{z_1(x+\xi)} \right] \phi(z_1) N_c \left[\Theta(1 \geq x \geq \xi) \phi\left(\frac{x-\xi}{1-\xi}\right) - \Theta(-\xi \geq x \geq -1) \phi\left(\frac{1+x}{1-\xi}\right) \right], \quad (7)$$

to be interpreted as a convolution $M = TDA \otimes CF \otimes DA$. The TDA is defined in the usual GPD kinematics, with skewedness $\xi = Q_1^2/(2s - Q_1^2)$ and momentum fractions along $n_2 = \frac{p_2}{1+\xi}$. This factorisation (Fig.5) is proven at Born order. First, one computes perturbatively the TDA $\gamma_L^* \rightarrow \rho_L^0$ defined

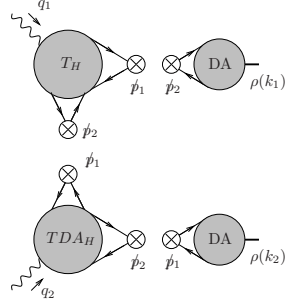


Figure 5: Factorization of the amplitude in terms of a TDA.

by a bilocal correlator. Q_2^2 being hard, the

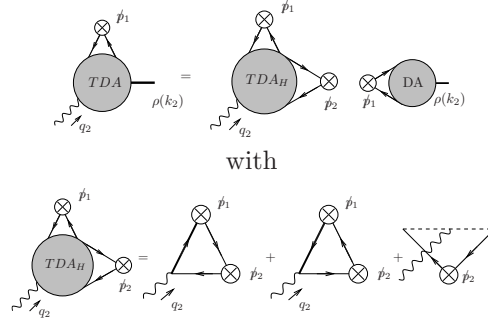


Figure 6: Perturbative TDA factorization.

TDA factorizes (Fig.6). To satisfy gauge invariance, a QED Wilson line is included (last term of Fig.6). Second, the Hard term is computed at Born order (see Fig.7). These two results combine according to (7).

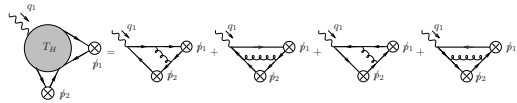


Figure 7: Hard part T_H at lowest order.

3 The high energy limit

QCD in the perturbative Regge limit is governed by gluons. BFKL enhancement is expected to be important at large rapidity. The exclusive process (1) tests this limit [10, 11, 12], for both Q_i^2 *hard* and of the *same order*. For $s_{\gamma^* \gamma^*} \gg -t$, Q_1^2, Q_2^2 , we rely on the impact representation which reads

$$\mathcal{M} = \frac{is}{16\pi^4} \int \frac{d^2 \underline{k}}{\underline{k}^2 (\underline{r} - \underline{k})^2} \mathcal{J}^{\gamma_{L,T}^* (q_1) \rightarrow \rho_L^0(k_1)}(\underline{k}, \underline{r} - \underline{k}) \\ \times \mathcal{J}^{\gamma_{L,T}^* (q_2) \rightarrow \rho_L^0(k_2)}(-\underline{k}, -\underline{r} + \underline{k})$$

at Born order. The impact factors $\mathcal{J}^{\gamma_{L,T}^*}$ are rational functions of the transverse momenta $(\underline{k}, \underline{r})$. The 2-d integration is treated analytically, through conformal transformations. The integrations over z_1 and z_2 (hid-

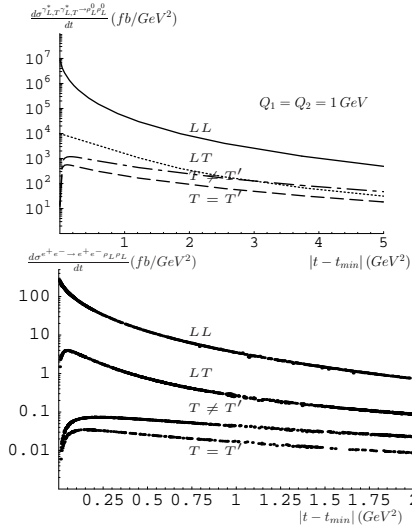


Figure 8: $\gamma_{L,T}^* \gamma_{L,T}^* \rightarrow \rho_L^0 \rho_L^0$ (up) and $e^+ e^- \rightarrow e^+ e^- \rho_L^0 \rho_L^0$ (down) differential cross-sections.

den in \mathcal{J}) are performed numerically. Cross-sections are strongly peaked at small Q_i^2 and t , and longitudinally polarized photons dominates (Fig.8up). The non-forward Born order cross-section for $e^+ e^- \rightarrow e^+ e^- \rho_L^0 \rho_L^0$ is obtained using the equivalent photon approximation. Defining y_i as the longitudinal momentum fractions of the bremsstrahlung photons, one finds that $\sigma^{e^+ e^- \rightarrow e^+ e^- \rho_L \rho_L}$ gets its main contribution from the low y and Q^2 region, which is the very forward region. At ILC, $\sqrt{s}_{e^+ e^-} = 500$ GeV, with 125 fb^{-1} per year. The measurement seems feasible since each detector design includes a very forward electromagnetic calorimeter with tagging angle for outgoing leptons down to 5 mrad. Fig.8down displays our results within the Large Detector Concept. We obtain

$\sigma^{tot} = 34.1 \text{ fb}$ and $4.3 \cdot 10^3$ events per year. The LL BFKL enhancement is enormous but not trustable, since it is well known that NLL BFKL is far below LL. Work to implement *resummed* LL BFKL effects [13] is in progress, with results in accordance with the NLL based one [14]. The obtained enhancement is less dramatic (~ 5) than with LL BFKL, but *still visible*.

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